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Naval Undersea Warfare Center Division  
Newport, Rhode Island



**A FAST MAXIMUM LIKELIHOOD  
MULTIDIMENSIONAL SEARCH ALGORITHM**

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15 September 1998

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Report Documentation Page			Form Approved OMB No. 0704-0188		
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1. REPORT DATE <b>15 SEP 1998</b>		2. REPORT TYPE <b>Technical Memo</b>		3. DATES COVERED <b>15-09-1998 to 15-09-1998</b>	
4. TITLE AND SUBTITLE <b>A Fast Maximum Likelihood Multidimensional Search Algorithm</b>			5a. CONTRACT NUMBER		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S) <b>Craig MacInnes</b>			5d. PROJECT NUMBER <b>C600068</b>		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Naval Undersea Warfare Center Division,1176 Howell Street,Newport,RI,02841</b>			8. PERFORMING ORGANIZATION REPORT NUMBER <b>TM 980145</b>		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) <b>Office of Naval Research</b>			10. SPONSOR/MONITOR'S ACRONYM(S) <b>ONR 321</b>		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S)		
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES <b>NUWC2015</b>					
14. ABSTRACT <b>This memorandum examines an application of the least-squares approximation of certain structured matrixes to a new algorithm for the maximum likelihood (ML) estimation of directions of arrival (DOAs) and frequencies of sinusoids. The method involves reordering the steps in the standard ML search algorithm so that the majority of the calculations are made prior to execution of the multidimensional search. The computational efficiency is due to the fact that only scalar operations are performed during the search. This new fast maximum likelihood estimation (FMLE) algorithm is more than one order of magnitude faster than the standard direct-search ML method.</b>					
15. SUBJECT TERMS <b>VLF; Very Low-Frequency Direction Finding Project; search algorithm; FMLE; fast maximum likelihood estimation</b>					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>	18. NUMBER OF PAGES <b>13</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

## **ABSTRACT**

This memorandum examines an application of the least-squares approximation of certain structured matrixes to a new algorithm for the maximum likelihood (ML) estimation of directions of arrival (DOAs) and frequencies of sinusoids. The method involves reordering the steps in the standard ML search algorithm so that the majority of the calculations are made prior to execution of the multidimensional search. The computational efficiency is due to the fact that only scalar operations are performed during the search. This new fast maximum likelihood estimation (FMLE) algorithm is more than one order of magnitude faster than the standard direct-search ML method.

## **ADMINISTRATIVE INFORMATION**

The work described in this memorandum was funded under NUWC Division Newport Job Order No. C600068, "VLF (Very Low-Frequency) Direction Finding Project," principal investigator Norman L. Owsley (Code 2123). The sponsoring activity is the Office of Naval Research, program managers Kenneth Dial and Nancy Harned (ONR 321).

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# A Fast Maximum Likelihood Multidimensional Search Algorithm

## 1. INTRODUCTION

The maximum likelihood (ML) estimation method is not always the method of choice for many applications because of the complexity of the calculations involved. This memorandum describes a more efficient computational method that will allow ML estimation to be used for a wider class of problems than at present.

The fast maximum likelihood estimation (FMLE) algorithm introduced here can be applied to two problems: direction-of-arrival (DOA) estimation and sinusoidal frequency estimation. Under mild restrictions on the statistics of the noise, these problems are shown to be equivalent to a structured least-squares problem. An estimate of a noise-corrupted matrix of data can be improved by taking advantage of prior knowledge concerning its underlying structure (for example, the underlying Toeplitz structure of a covariance matrix).<sup>1,2,3</sup> Another type of estimate is one that is both structured and low rank.<sup>4,5</sup>

In this study, the ML matrix estimate is required to have a specific rank, which is equal to the number,  $N$ , of sources present. In addition, the estimate is required to have a column space spanned by a set of  $N$  basis vectors that have a specific structure. In this manner, the ML problem may be thought of as a type of structured low-rank approximation.

In section 2, two structured least-squares approximation problems are introduced and the solutions are written in a form involving the maximization of a single expression. Section 3 examines applications of ML estimation to the determination of DOAs and to the estimation of frequencies of sinusoids in noise. In section 4, the FMLE method is developed. After the results of the simulations are given in section 5, section 6 concludes the memorandum.

## 2. FORMULATION OF TWO LEAST-SQUARES PROBLEMS

In this memorandum, the columns  $\{d_1, \dots, d_r\}$  of the matrix  $D$ , over which each search is to be made, are of a special form. Each column of  $D$  can be written as an  $L$ -vector:

$$v = [1 \ z \ z^2 \ \dots \ z^{L-1}]^T.$$

The vector,  $v$ , will be called a *Fourier vector*, because when  $z = e^{i\theta}$ , the inner product  $v^H w$  can be interpreted as the discrete Fourier transform (DFT) of  $w$ , evaluated at  $z$ . For DOA problems, Fourier vectors are also referred to as steering vectors. The first least-squares problem is presented next:

**Problem 1**— Let  $A$  be a complex  $L \times s$  matrix. Find an  $L \times r$  matrix,  $D = [d_1, \dots, d_r]$ , with distinct Fourier vectors as columns and an  $r \times s$  matrix,  $\rho$ , such that the following is minimized:

$$\arg \min_{D, \rho} \|A - D\rho\|_F. \quad (1)$$

We then have

$$\arg \min_{D, \rho} \|A - D\rho\|_F^2 = \arg \min_{D, \rho} \text{trace} \left( A^H A - A^H D\rho - \rho^H D^H A + \rho^H D^H D\rho \right). \quad (2)$$

Using the projection theorem,<sup>6</sup> the columns of the error matrix  $A - D\rho$  of the approximation are orthogonal to the column space in which the solution lies. Therefore, the orthogonal projection of the error onto the column space of  $D$  will be zero:

$$D(D^H D)^{-1} D^H (A - D\rho) = 0.$$

Since  $D$  is a Vandermonde matrix with distinct columns,  $D^H D$  is nonsingular.<sup>7</sup> We then have

$$\rho = (D^H D)^{-1} D^H A.$$

From equation (2), it follows that

$$\arg \min_{D, \rho} \|A - D\rho\|_F^2 = \arg \max_D \text{trace} \left( A^H D (D^H D)^{-1} D^H A \right). \quad (3)$$

**Problem 2**— Let  $A$  be a complex  $L \times L$  Hermitian matrix. Find an  $L \times r$  matrix,  $D$ , where the columns,  $d_i, i = 1, \dots, r$ , of  $D$  are Fourier vectors and an  $r \times r$  matrix,  $\mu$ , such that the following is minimized:

$$\arg \min_{D, \mu} \|A - D\mu D^H\|_F. \quad (4)$$

In a similar manner to the previous case, this leads to the same minimization problem:

$$\arg \min_{D, \mu} \|A - D\mu D^H\|_F^2 = \arg \max_D \text{trace} \left( A^H D (D^H D)^{-1} D^H A \right), \quad (5)$$

where  $\mu = (D^H D)^{-1} D^H A D (D^H D)^{-1}$ . Let  $\widehat{D}$  be any orthonormal set of vectors spanning the same column space as  $D$ . Due to the uniqueness of an orthogonal projection matrix, we have

$$D(D^H D)^{-1} D^H = \widehat{D} \widehat{D}^H.$$

We can then further simplify the right-hand sides of equations (3) and (5) to

$$\arg \max_D \|\widehat{D}^H A\|_F^2. \quad (6)$$

The least-squares estimates considered above and the ML estimation methods to be considered in the next section are both related to a third type of estimate, which is the minimum variance unbiased estimate. If the noise is Gaussian and the error covariance is the identity matrix scaled by the noise variance, then these three estimates are equivalent.<sup>6,8</sup> We assume that these restrictions hold.

### 3. DEVELOPMENT OF THE MAXIMUM LIKELIHOOD ESTIMATION EQUATIONS

#### 3.1 ESTIMATION OF DIRECTIONS OF ARRIVAL

This section examines an application of ML estimation to the determination of DOAs in an array processing scenario, where the signals may be temporally coherent. This case cannot be dealt with directly by subspace-based methods without further processing<sup>9</sup> or array partitioning.<sup>10</sup>

The DOA problem treated here arises from  $M$  narrowband farfield signals impinging on a linear array of  $L$  equally spaced sensors. The sensor outputs are subject to additive zero-mean white Gaussian noise. The equation describing this scenario is

$$x(k) = B(k)s(k) + n(k), \quad k = 1, 2, \dots, \quad (7)$$

where  $x(k)$ , called a data snapshot, is an  $L$ -vector of array outputs at sampling time  $k$ . The columns of  $B(k) = [b_1(k) \ b_2(k) \ \dots \ b_M(k)]$  are the steering vectors at time  $k$ :

$$b_i(k) = [1 \exp(2\pi j \beta \cos(\theta_i(k))) \ \dots \ \exp(2\pi j (L-1) \beta \cos(\theta_i(k)))]^T, \quad i = 1, \dots, M.$$

Here,  $\theta_i$  is the  $i$ th bearing relative to broadside and  $\beta$  is the interelement spacing of the array in wavelengths. The value of  $M$  can be estimated, for example, by using the model-based approach given in reference 11. The elements of the signal vector,  $s(k)$ , are complex exponentials:

$$s_i(k) = c_i(k) \exp(j\varphi_i(k)), \quad i = 1, \dots, M, \quad k = 1, 2, \dots$$

Let  $E\{x\}$  be the expected value of the random vector,  $x$ . We then have

$$C \stackrel{\text{def}}{=} E\{xx^H\} = E\{(Bs + n)(Bs + n)^H\} = B E\{ss^H\} B^H + E\{nn^H\} = BSB^H + \sigma^2 I.$$

The ML estimate  $\hat{C}$  of the estimated covariance matrix is

$$\hat{C} \stackrel{\text{def}}{=} \frac{1}{N} \sum_{i=1}^N x(i)x(i)^H. \quad (8)$$

It is assumed here that the source DOAs are constant during the time period over which the average in equation (8) is taken. The solution of equation (7) corresponding to the concentrated likelihood function in reference 8 is

$$\arg \max_D \left( x(i)^H \left( D(D^H D)^{-1} D^H \right) x(i) \right), \quad (9)$$

where  $x(i)$  is the data snapshot obtained at time  $i$ . The ML estimation of the true covariance matrix,  $C$ , can be obtained by taking  $N$  data snapshots and averaging the expression in equation (9) over the following terms:

$$\begin{aligned}
& \frac{1}{N} \sum_{i=1}^N \arg \max_D \text{trace} \left( x(i)^H D (D^H D)^{-1} D^H x(i) \right) \\
&= \arg \max_D \text{trace} \left( D (D^H D)^{-1} D^H \frac{1}{N} \sum_{i=1}^N x(i) x(i)^H \right) \\
&= \arg \max_D \text{trace} \left( D (D^H D)^{-1} D^H \hat{C} \right) \\
&= \arg \max_D \text{trace} (U^H D (D^H D)^{-1} D^H U) ,
\end{aligned}$$

where  $UU^H$  is the Cholesky factorization of  $\hat{C}$ .

### 3.2 ESTIMATION OF FREQUENCIES OF SINUSOIDS

Consider taking  $L$  consecutive samples from a time series containing a weighted sum of  $M$  exponentials in Gaussian white noise:

$$y(n) = \sum_{i=1}^M \alpha_i \exp(2\pi j f_i n) + w(n), \quad n = 1, \dots, L.$$

The ML method applied to this problem<sup>12</sup> involves the minimization of the following error:

$$E = \sum_{n=1}^L \left| y(n) - \sum_{i=1}^M \hat{\alpha}_i \exp(2\pi j \hat{f}_i n) \right|^2,$$

where the  $\alpha_i$ 's and  $f_i$ 's are to be estimated. Let  $Y = [y(1) \cdots y(L)]^T$ . The least-squares solution then is

$$\arg \max_D \left( Y^H D (D^H D)^{-1} D^H Y \right), \quad (10)$$

from which a square root can be calculated:  $(D^H D)^{-1} = F^2$ . It follows that the columns of  $DF$  are orthonormal. With this result, equation (10) can be rewritten as

$$\arg \min_D \|F^H D^H Y\|_F^2. \quad (11)$$

An alternate orthonormalization for  $D$  is considered in the next section.

### 4. DERIVATION OF THE FMLE ALGORITHM

We now describe a way to take advantage of the simplified form of the solution in the expression

$$\arg \max_D \|\hat{D}^H A\|_F^2. \quad (12)$$



In the following discussion, assume that  $A$  is a given  $L \times Q$  matrix and that  $D = [d_1 \cdots d_r]$  is an  $L \times r$  matrix ( $r \leq Q$ ), with distinct Fourier vectors as columns.

The columns of  $D$  are to be selected from a given  $L \times K$  matrix  $B$  of Fourier vectors, and the multidimensional search is performed over all subsets of  $r$  distinct columns of  $B$ . We seek an alternative to using the factor,  $F$ , in equation (11) for orthonormalizing the columns of  $D$ . Since the Frobenius norm is unitarily invariant, all orthonormal bases for the column space of  $D$  are equivalent, with respect to the value of the expression in equation (12). We will use the Gram-Schmidt procedure to accomplish the orthonormalization of the columns of  $D$ .

In order to calculate the inner products  $d_i^H d_j$  between the Fourier vectors  $d_i$  and  $d_j$ , we define the matrix  $[\gamma]$ :

$$(B^H B)_{ij} \stackrel{\text{def}}{=} \gamma_{ij}, i, j = 1, \dots, K.$$

It will be assumed that the columns of  $B$  are normalized;  $(B^H B)_{ii} = 1$ ,  $i = 1, \dots, K$ . It is easily shown that  $B^H B$  is Toeplitz, as well as Hermitian. Therefore, only the first row of  $B^H B$  needs to be formed. Consider two Fourier vectors,  $d_n$  and  $d_m$ , where  $d_n$  is

$$d_n \stackrel{\text{def}}{=} [1 \exp(j\mu n) \cdots \exp(j\mu(L-1)n)]^T.$$

We now have

$$\gamma_{nm} \stackrel{\text{def}}{=} d_n^H d_m = \frac{1 - \exp(-j\mu(n-m)L)}{1 - \exp(-j\mu(n-m))}, \quad (13)$$

where  $j \stackrel{\text{def}}{=} \sqrt{-1}$ . We also define the matrix  $[\alpha] \stackrel{\text{def}}{=} B^H A$ , where

$$\alpha_{nm} \equiv (B^H A)_{nm}, \quad n = 1, \dots, K; \quad m = 1, \dots, Q.$$

The inner products in this matrix-matrix product involve the unorthogonal Fourier vectors. We next describe the search algorithm. To illustrate the overall procedure, let  $r = 2$ . We then execute the two nested loops as follows: for  $i = 1, \dots, M-1$ ; for  $j = i+1, \dots, M$ .

Consider the entries of the matrix  $D^H A$ , where  $D = [d_i \ d_j]$ . The maximization is

$$\arg \max_{D^\dagger} \|D^\dagger{}^H A\|_F^2,$$

where  $D^\dagger$  is any orthonormalization of the columns of  $D$ . Given  $i$  and  $j$ , the sum of the squared magnitudes of the terms in the first row of  $D^H A$  is

$$\sum_{k=1}^Q |d_i^H a_k|^2, \quad (14)$$

where  $a_k$  is the  $k$ th column of  $A$ . For the second row, we must use the orthonormalized values for  $d_j$  given  $d_i$ :

$$\hat{d}_j \stackrel{\text{def}}{=} (d_j - (d_i^H d_j) d_i) / \beta_{ij}, \quad j = 1, \dots, r,$$

where

$$\beta_{ij} \stackrel{\text{def}}{=} \|\hat{d}_j\|_2 = \|d_j - (d_i^H d_j) d_i\|_2$$

is the normalizing factor. The norm  $\beta_{ij}$  of  $\hat{d}_j$  can be found from the matrix  $[\gamma]$  since

$$\begin{aligned} \beta_{ij}^2 = \hat{d}_j^H \hat{d}_j &= (d_j^H - \overline{(d_i^H d_j)} d_i^H) (d_j - (d_i^H d_j) d_i) \\ &= 1 - |\gamma_{ij}|^2 \end{aligned} \quad (15)$$

for  $i \neq j$ . In the general case for  $r > 2$ , equation (15) becomes

$$\begin{aligned} \hat{d}_j^H \hat{d}_j &= \left( d_j - \sum_{i=1}^{j-1} (d_i^H d_j) d_i \right)^H \left( d_j - \sum_{i=1}^{j-1} (d_i^H d_j) d_i \right) \\ &= 1 - 2 \sum_{i=1}^{j-1} |\gamma_{ij}|^2 + \sum_{n=1}^{j-1} \sum_{m=1}^{j-1} \bar{\gamma}_{nj} \gamma_{mj} \gamma_{nm} . \end{aligned}$$

The sum of the squared magnitudes of the elements in the second row for  $D^{\dagger H} A$ , given  $d_i$ , is then

$$\sum_{k=1}^Q |\hat{d}_j^H a_k|^2 . \quad (16)$$

Consider the following term from equation (16):

$$\begin{aligned} \hat{d}_j^H a_k &= ((d_j^H - (d_i^H d_j) d_i^H) / \beta_{ij}) a_k \\ &= (d_j^H a_k - \bar{\gamma}_{ij} d_i^H a_k) / \beta_{ij} \\ &= (\alpha_{jk} - \bar{\gamma}_{ij} \alpha_{ik}) / \beta_{ij}, \quad k = 1, \dots, Q . \end{aligned} \quad (17)$$

In this key equation, the computational savings to be realized are due to the fact that all the individual terms are scalars, and each term can be calculated before the execution of the nested loops constituting the search. Combining equations (14), (16) and (17), the expression for  $\|D^{\dagger H} A\|_F^2$ , where  $D = [d_i \ d_j]$ , is

$$\begin{aligned} \|D^{\dagger H} A\|_F^2 &= \sum_{k=1}^Q |d_i^H a_k|^2 + \sum_{k=1}^Q |\hat{d}_j^H a_k|^2 \\ &= \sum_{k=1}^Q |\alpha_{ik}|^2 + \sum_{k=1}^Q |\alpha_{jk} - \bar{\gamma}_{ij} \alpha_{ik}|^2 / \beta_{ij}^2 . \end{aligned} \quad (18)$$

It can be seen from the simulations that this approach results in over an order of magnitude gain in efficiency when compared with the standard ML search procedure. In summation of this section, the newly developed FMLE algorithm for an  $r$ -dimensional search procedure involves the following steps:

1. Compute the distinct entries of  $B^H B$  at the beginning of the analysis, using equation (13).

2. Compute  $B^H A$  before entering the nested loops. This can be done efficiently with an update procedure whenever a sliding window or an exponential window is used. If the Fourier vectors in the columns of  $B$  are equally spaced in angle,  $B^H A$  can be found by computing the DFTs of the columns of  $A$  and summing the squared magnitudes of the elements in each row.
3. Conduct the  $r$ -level multidimensional search.

The computational complexity of FMLE is  $O(K^r)$ , where there is an  $r$ -level multidimensional search over a set of  $K$  Fourier vectors. The standard ML search for this problem is also of  $O(K^r)$  complexity, but the constant is much larger.

## 5. SIMULATIONS

In the first simulation, as shown in figure 1, the performance of the ML and FMLE algorithms is compared with the number of array elements. Since there are only scalar operations performed within the FMLE search, the FMLE complexity is of lower order than the ML complexity.

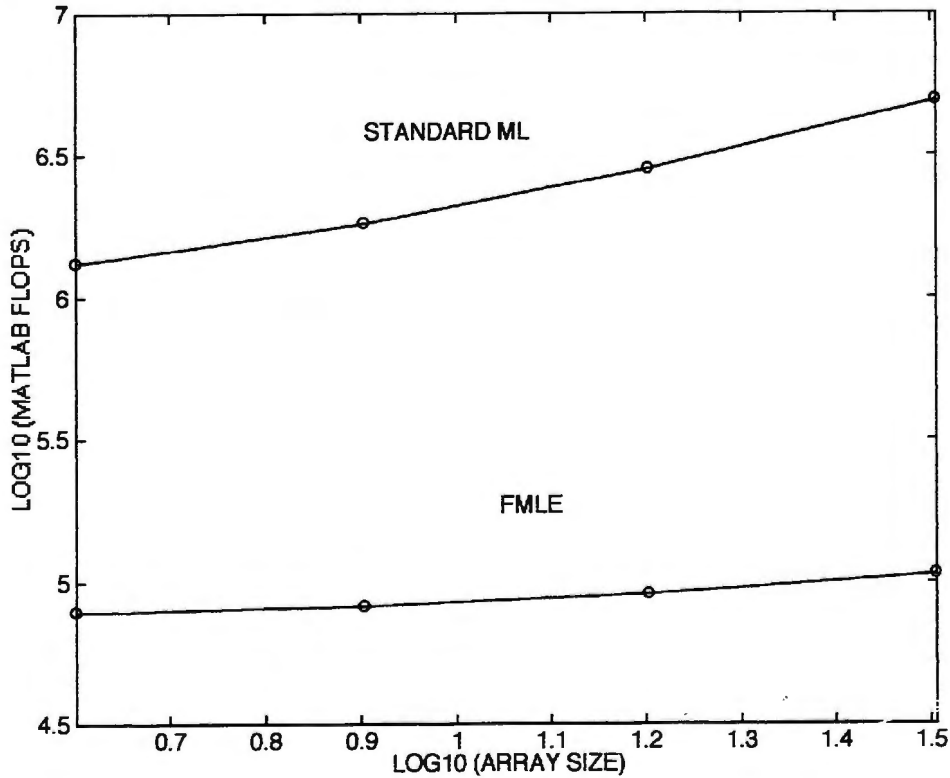


Figure 1: *Comparison of Standard ML and FMLE Performance Versus Array Size.*

The second simulation, as shown in figure 2, compares the ML and FMLE performance versus the number of sources present, i.e., the dimension of the multidimensional search.

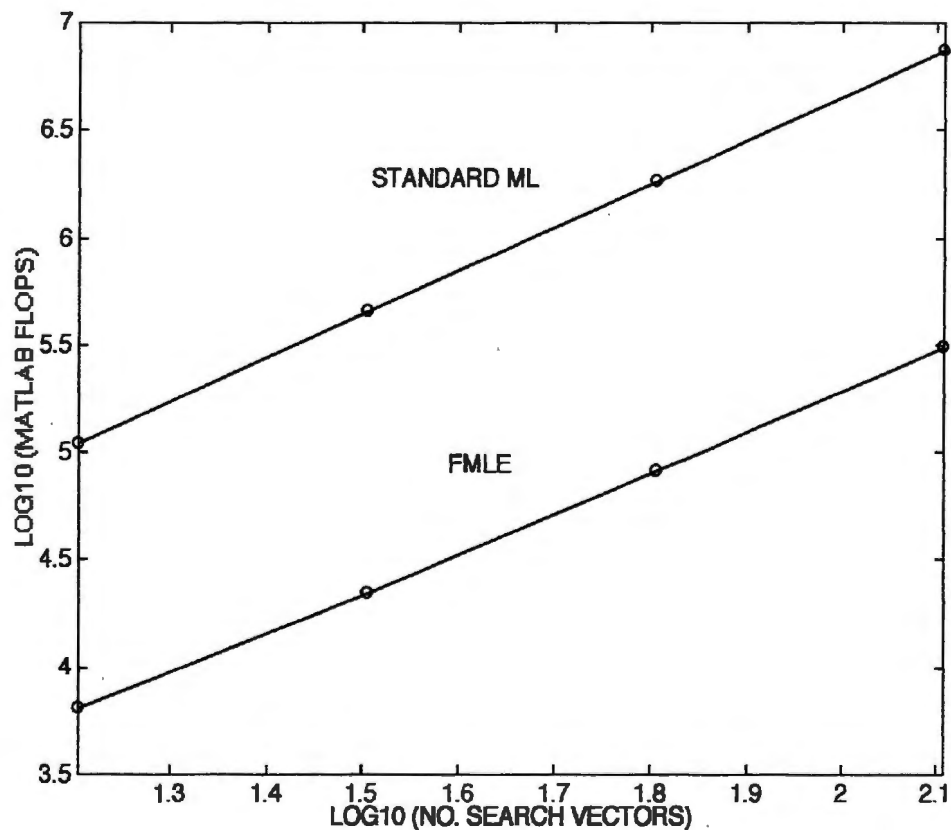


Figure 2: *Complexity Versus Dimension of Search.*

## 6. CONCLUSIONS

It has been shown that the ML estimation of DOAs or frequencies of sinusoids can be formulated as a least-squares matrix approximation problem, under mild restrictions on the noise structure. The efficiency of the FMLE algorithm is made possible by the performance of all nonscalar calculations before the execution of the nested loops constituting the multidimensional search. This approach results in an order of magnitude speed-up in the ML search.

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